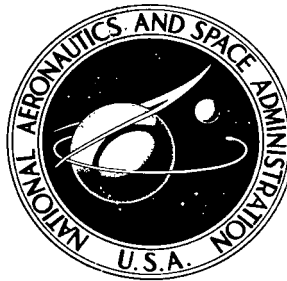


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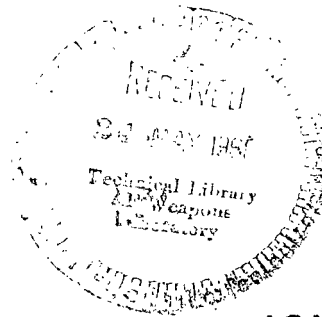


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ASPECT DETERMINATION IN LUNAR SHADOW ON EXPLORER 35

by Harold E. Taylor

*Goddard Space Flight Center
Greenbelt, Md.*

ERRATUM

NASA Technical Note D-4544

Aspect Determination in Lunar Shadow on Explorer 35

by Harold E. Taylor

Completed 24 Sep 69 - SW

The captions of Figures 2 and 3 were interchanged. The corrected versions appear below.



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ASPECT DETERMINATION IN LUNAR
SHADOW ON EXPLORER 35

By Harold E. Taylor

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Greenbelt, Md.

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ABSTRACT

The Explorer 35 spacecraft experiences a small, temporary secular decrease in spin period (approximately 2 parts in 1000) during every pass through the lunar shadow. Because of this, the pseudo-sun pulse, generated when the spacecraft is in the shadow, no longer accurately indicates the sunward direction. This report describes an empirical method of fitting a spin period of the form $a + b \exp(-t/t_0)$ to the aspect information available before and after the optical shadow (t is time since the spacecraft entered the shadow, and a , b , and t_0 are constants to be determined). The resulting aspect information is estimated to be within ± 10 degrees of the true direction.

CONTENTS

Abstract.	ii
INTRODUCTION.	1
STATEMENT OF THE PROBLEM.	1
COMPUTATION OF SATELLITE'S ROTATIONS.	4
NUMERICAL EVALUATION OF PARAMETERS AND THEIR APPLICATION	8

ASPECT DETERMINATION IN LUNAR SHADOW ON EXPLORER 35

by

Harold E. Taylor
Goddard Space Flight Center

INTRODUCTION

Explorer 35, injected into selenocentric orbit on July 22, 1967, experiences a decrease in its moment of inertia as it passes through the lunar shadow. This apparently is caused by the cooling of the spacecraft and, most probably, by inward contraction of the solar paddles. Because angular momentum is conserved, the spin period decreases slightly (a few parts per thousand), and the pseudo-sun pulse generated onboard the spacecraft no longer accurately indicates the direction of the sun. This report describes an empirical approach to solving the resulting satellite aspect problem.

STATEMENT OF THE PROBLEM

A variable spin period of the form

$$\tau = a + be^{-t/t_0} \quad (1)$$

is assumed, during the passage through shadow (a , b , and t_0 being fitted parameters to be determined). The spin periods (which are measured during the recovery following a passage through shadow) obey this type of exponential law (Figure 1). During the shadow, the absence of a true sun pulse precludes the accurate determination of the spin period. However, to the extent that the azimuth angle of the magnetic field vector in the satellite spin plane remains constant throughout the shadow, the magnetic field measurements themselves give an estimate of the satellite azimuth. A comparison of the apparent rotation calculated using the methods to be described is shown in Figure 2. This plot and others like it for subsequent shadow passes

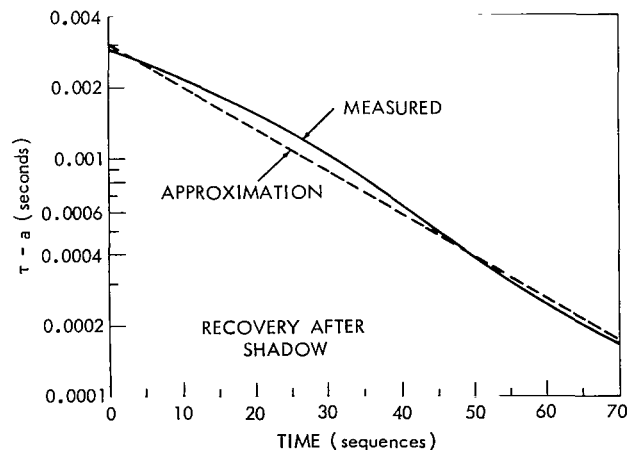


Figure 1—Observed $\tau - a$ versus time for the recovery of the spacecraft following the first shadow pass for which complete data were received (solid line); compared with this is the approximation of an exponential time-dependence (dashed line).

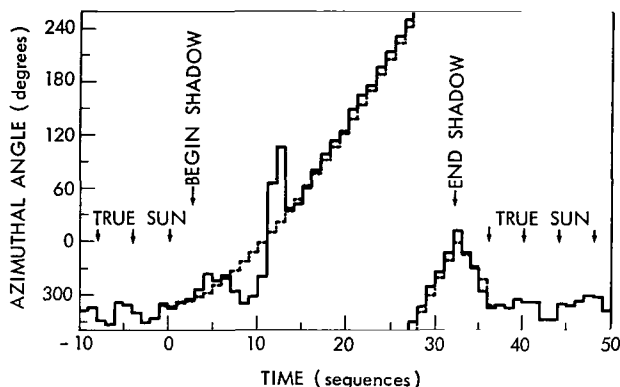


Figure 2—Change of apparent azimuth of magnetic field observed (solid line) during same shadow pass as in Figure 1; compared is the predicted change (dashed line) calculated by assuming that the magnetic field did not change its direction.

tered once every four sequences. This enables one to calculate accurately the azimuthal orientation of the spacecraft about the spin axis at any time.

Such an accurate determination of the spacecraft azimuth requires the calculation of a refined spin period. This is done by calculating the time between successive sun times and dividing by the integral number of revolutions made by the spacecraft. The number of revolutions N is calculated from the raw spin period. Thus the refined spin period is given by

$$\tau = \frac{C_0 + C_2 - C_1}{N}, \quad (2)$$

where

- C_0 = the time between successive sun reference times,
- C_1 = sun time at beginning of interval,
- C_2 = sun time at end of interval, and
- N = integral number of revolutions between sun sightings.

N in turn is given by

$$N \pm \Delta = \frac{C_0 + C_2 - C_1}{S}, \quad (3)$$

where S is the raw spin period, and Δ is a fraction. The fraction Δ will be less than 0.5 as long as the time C_0 corresponds to less than approximately 40 sequences. Thus N is found easily by rounding the quantity $(N + \Delta)$ to the nearest integer.

attest to the accuracy of the calculations, using a time-dependent spin period of the form in Equation 1.

To help clarify the method of analyzing the spacecraft aspect in the shadow, a brief description of the pertinent facts of spacecraft operation is necessary. The direction of the sun relative to the spacecraft coordinate system is measured once every four telemetry sequences (1 sequence = 81.808 seconds).

This is measured by counting the time from a fixed reference point in the telemetry sequence to the next "see sun" pulse from the onboard sun sensor.* This time (referred to as the "sun time") and the time between consecutive sun pulses (the raw spin period) are telemetered once every four sequences.

*AIMP (IMP-D) Technical Summary Description, GSFC Document X-724-67-87, March 1967.

If the spin period remained constant through the shadow, it would be a simple matter to determine accurately the azimuthal orientation of the spacecraft. Actually the spacecraft spins up in the shadow; and, by the end of the shadow, a large error (of the order of 360 degrees) has accumulated in the satellite orientation computed from the optical aspect data. It is this error that must be corrected.

The time when the spacecraft enters the shadow is measured to the nearest half sequence, except every fourth sequence when it is accurate only to the nearest sequence. This information is telemetered in a sun flag, which indicates whether the sun sensor is seeing the sun or not.

Thus the situation is as follows: The spin period and sun direction are known accurately at the time of the last sun sighting before the shadow. Then there follow 0 to 4 sequences of measurements before the satellite enters the shadow and the spin period begins to change. Upon entering the shadow, the spacecraft begins to spin up, and the spin period begins to decrease. This decrease continues until the spacecraft again enters the sunlight whereupon it begins to despin again; and the spin period begins to increase, finally approaching its preshadow value. After leaving the shadow, there will be 0 to 4 sequences before true solar aspect data are obtained. The total fractional change in the spin period is approximately 0.0015 (0.15 percent) over the period of the shadow—typically 40 minutes (35 sequences).

With this understanding of the problem, calculating the three free parameters in Equation 1 is fairly straightforward. The following parameters are used (Figure 3 illustrates their significance):

- Δt_i = time between the last true sun pulse and the beginning of the shadow,
- Δt_s = duration of the shadow,
- Δt_f = time between shadow end and next true sun time,
- τ_i = spin period before shadow,
- τ_1 = first spin period measured after shadow,
- τ_2 = second spin period measured after shadow,
- C_i = last sun time before shadow,
- C_f = first sun time after shadow, and
- S = raw spin period.

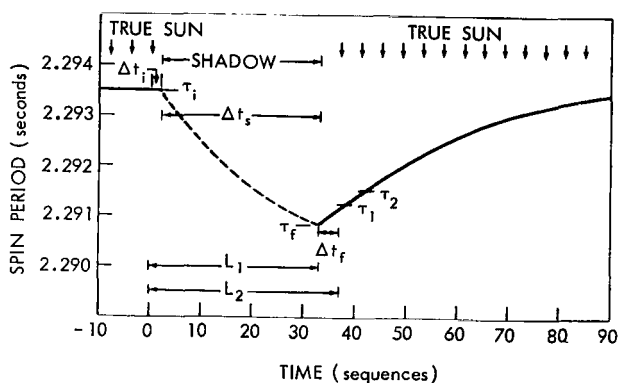


Figure 3—Spin period versus time for the same shadow pass as in Figure 1; the solid part of the curve is derived from measured spin periods, and the dashed portion in the shadow is the approximation of Equation 1 with appropriate values of the constants a , b , and t_0 substituted.

In addition the following parameters will be used:

- τ_f = spin period at the end of the shadow,
- $R(t)$ = actual number of revolutions of the spacecraft since last true sun before shadow (time "zero" on Figure 3),
- R_0 = $R(t = \Delta t_i + \Delta t_s + \Delta t_f)$ = total number of spins made between true sun pulses, and
- N_0 = integral number of rotations of spacecraft between true sun pulses.

Convenient units are as follows:

- Δt 's and τ 's in seconds,
- C_i , C_f , and S in 800-Hz clock counts,
- R , R_0 , and N in revolutions.

It is often useful to express the Δt 's in units of sequence length (81.808 seconds); t_0 also will be measured in units of sequence length.

COMPUTATION OF SATELLITE'S ROTATIONS

The problem now becomes one of fitting a spin period of the form in Equation 1 to the satellite data. There are three free parameters in one; three independent equations are required to determine these parameters. Two are obvious, stating that the spin period before and after the shadow must be equal to that measured. The third states that the integrated number of revolutions between the true sun times be equal to that achieved by the spacecraft. To this end, τ_f and R_0 must be determined since they are not measured directly.

First, τ_f may be found by using a linear interpolation from τ_1 and τ_2 at the end of the shadow. (Earlier work also fits these data to an exponential to find τ_f ; but this was found to be unnecessary, the linear approximation being more than accurate enough for the short interval involved.) Thus (for Δt_f in units of the sequence length)

$$\tau_f = \tau_1 - \left(\frac{\tau_2 - \tau_1}{4} \right) (\Delta t_f + 2) . \quad (2)$$

A little thought will reveal that R_0 is given by

$$R_0 = N_0 - \frac{C_f - C_i}{S} , \quad (3)$$

where N_0 is a constant to be determined; it may be found by integration if the spin period is known in terms of time. In this work, the spin period is assumed to be given by

$$\tau(t) = \begin{cases} \tau_i, & t \leq \Delta t_i \\ a + be^{-[(t - \Delta t_i)/t_0]}, & \Delta t_i < t \leq L_1 \\ \tau_f + \left(\frac{\tau_2 - \tau_1}{4}\right) (t - \Delta t_s - \Delta t_i), & L_1 < t \leq L_2 \end{cases} \quad (4)$$

where $L_1 = \Delta t_i + \Delta t_s$, and $L_2 = L_1 + \Delta t_f$. Now R_0 may be obtained from:

$$\begin{aligned} R_0 &= \int_0^{L_2} \frac{dt}{\tau(t)} \\ &= \frac{\Delta t_i}{\tau_i} + \frac{\Delta t_s}{a} - \frac{t_0}{a} \ln \left(\frac{a + be^{-\Delta t_s/t_0}}{a + b} \right) + \tau_f + \left(\tau_2 - \tau_1 \right) \frac{\Delta t_f}{4}. \end{aligned} \quad (5)$$

Since initially the values of a , b , and t_0 are unknown, we still do not know the value of R_0 . However, the present goal is to obtain N_0 ; therefore, it is sufficient to know R_0 within ± 0.5 revolution. The following approximate values of a , b , and t_0 give R_0 within ± 0.1 revolution:

$$a \approx \tau_f - 0.0001 \text{ second},$$

$$b = \tau_i - a, \approx (\tau_i - \tau_1) + 0.0001 \text{ second, and}$$

$$t_0 = 24 \text{ sequences} \approx 1963 \text{ seconds}.$$

Thus by rounding R_0 to the nearest integer, N_0 is determined. This is exact if the estimate of R_0 was within ± 0.5 revolution; then Equation 3 may be used to determine R_0 accurately. The accuracy of N_0 is easily verified since an error shows up in the magnetic field data as an apparent rotation of one or more complete revolutions. When the foregoing information has been derived, it is a fairly straightforward matter to establish the three conditions that determine the three unknown parameters in Equation 1: a , b , and t_0 .

The first two equations are obtained by requiring that the spin period be continuous at the beginning and end of the shadow. When the satellite enters the shadow, $t = \Delta t_i$ and $\tau = \tau_i$ so that

$$a + b = \tau_i. \quad (6)$$

At the end of the shadow, $t = L_1$ and $\tau = \tau_f$. Thus

$$a + be^{-(\Delta t_s/t_0)} = \tau_f. \quad (7)$$

The third and final condition is established by requiring that the predicted number of revolutions between the last real-sun pulse before the shadow and the first real-sun pulse after the

shadow is equal to the actual number of revolutions, R_0 . This requirement is expressed by Equation 5. Since some of the terms in Equation 5 involve only the known parameters (i.e., do not involve a , b , or t_0), it is useful to define another quantity R_s , the number of revolutions the spacecraft makes during the actual shadow:

$$R_s = R_0 - \frac{\Delta t_i}{\tau_i} - \frac{\Delta t_f}{\tau_f(\tau_2 - \tau_1)} \frac{\Delta t_f}{4} . \quad (8)$$

Then the third condition may be rewritten as

$$R_s = \frac{\Delta t_s}{a} - \frac{t_0}{a} \ln \left[\frac{a + b e^{-\left(\Delta t_s/t_0\right)}}{a + b} \right] . \quad (9a)$$

Since the change in the spin period during the shadow is observed to be only approximately 1/1000 of the spin period, it is easily seen that $b/a \sim 0.001 \ll 1$. Thus it simplifies the problem to expand the logarithmic term in Equation 9a in powers of b/a . This results in

$$R_s = \frac{\Delta t_s}{a} - \frac{t_0 b}{a^2} \left[1 - e^{-\left(\Delta t_s/t_0\right)} \right] , \quad (9b)$$

where only terms up to the first power of b/a have been retained.

The next step is to solve Equations 6, 7, and 9b simultaneously for a , b , and t_0 . From Equation 9b,

$$\frac{t_0 b}{a^2} e^{-\left(\Delta t_s/t_0\right)} = R_s - \frac{\Delta t_s}{a} + \frac{t_0 b}{a^2} . \quad (10)$$

From Equation 7,

$$\frac{t_0 b}{a^2} e^{-\left(\Delta t_s/t_0\right)} = \frac{t_0}{a^2} (t_f - a) ,$$

and from Equation 6, $b = \tau_i - a$. Substituting the last two expressions into Equation 10 yields

$$R_s - \frac{\Delta t_s}{a} + \frac{t_0}{a^2} (\tau_i - \tau_f) = 0 .$$

Upon rearrangement, this expression can be written

$$a^2 \frac{R_s}{\Delta t_s} - a + \frac{t_0}{\Delta t_s} (\tau_i - \tau_f) = 0 .$$

From Equations 6 and 7 it is seen that

$$e^{-(\Delta t_s/t_0)} = \frac{\tau_i - a}{\tau_f - a},$$

so that

$$\ln \left(\frac{\tau_i - a}{\tau_f - a} \right) = \frac{\Delta t_s}{t_0}$$

or

(11)

$$t_0 = \frac{\Delta t_s}{\ln \left(\frac{\tau_i - a}{\tau_f - a} \right)}.$$

Substituting the foregoing expression in Equation 10 yields

$$a^2 \frac{R_s}{\Delta t_s} - a + \frac{\tau_i - \tau_f}{\ln \left(\frac{\tau_i - a}{\tau_f - a} \right)} = 0. \quad (12)$$

It is possible to find an approximate solution to Equation 12 by use of Newton's method.* This is an iterative method for finding the roots of an equation of the form $f(X) = 0$. In essence, an initial estimate of the root X_0 is used to find an improved estimate X_1 from the equation

$$X_1 = X_0 - \frac{1}{f'(X_0)} \frac{df(X)}{dX}.$$

Thus, in the present problem,

$$f(a) = 0, \text{ where } f(a) = \frac{a^2 R_s}{\Delta t_s} - a + \frac{\tau_i - \tau_f}{\ln \left(\frac{\tau_i - a}{\tau_f - a} \right)}.$$

Then

$$\frac{df}{da} = \frac{2a R_s}{\Delta t_s} - 1 - \left[\frac{\tau_i - \tau_f}{\ln \left(\frac{\tau_i - a}{\tau_f - a} \right)} \right]^2 \frac{1}{(\tau_i - a)(\tau_f - a)}.$$

When a has been determined this way, it is a simple matter to use Equation 6 to determine b , and Equation 11 to obtain t_0 .

*Philip Franklin, "Methods of Advanced Calculus," pp 26-28, New York: McGraw-Hill, 1944.

Integration of Equation 1 yields the actual number of rotations of the satellite; i.e.,

$$R(t) = \begin{cases} \frac{t}{\tau_i} & t \leq \Delta t_i \\ R_1 + \frac{t - \Delta t_i}{a} - \frac{t_0 b}{a^2} \left[1 - \exp - \frac{(t - \Delta t_i)}{t_0} \right] & \Delta t_i \leq t \leq L_1 \\ R_2 + \frac{t - L_1}{\tau_f + \left(\frac{\tau_2 - \tau_1}{4} \right)} (t - L_1) & L_1 \leq t \leq L_2 \end{cases}$$

where $R_1 = \Delta t_i / \tau_i$, and

$$R_2 = R_1 + \frac{\Delta t_s}{a} - \frac{t_0 b}{a^2} \left[1 - \exp \left(- \frac{\Delta t_s}{t_0} \right) \right].$$

NUMERICAL EVALUATION OF PARAMETERS AND THEIR APPLICATION

An application of the method previously described to the pass shown in Figure 2 is useful for illustrative purposes. The input parameters are as follows:

$$\begin{aligned} \tau_i &= 2.2935 \text{ seconds,} \\ \tau_f &= 2.2909 \text{ seconds,} \\ \Delta t_s &= 30.5 \text{ sequences, and} \\ R_s &= 1088.675 \text{ revolutions.} \end{aligned}$$

The resulting values for a , b , and t_0 are

$$\begin{aligned} a &= 2.2898 \text{ seconds,} \\ b &= 0.003641 \text{ second, and} \\ t_0 &= 23.96 \text{ sequences.} \end{aligned}$$

With the additional information that

$$\begin{aligned} \tau_1 &= 2.29125, \\ \tau_2 &= 2.29151, \\ \Delta t_i &= 1.5, \\ \Delta t_f &= 4.0, \end{aligned}$$

and a knowledge of the actual spin period, the correction angles can be calculated as follows. If

$$R_u(t) = \int_0^t \frac{dt}{\tau_u},$$

where τ_u is the spin period used in the initial analysis, then the angle between the apparent sun direction (represented by the pseudo-sun pulse) and the true sun direction is given by

$$\begin{aligned}\psi(t) &= [R(t) - R_u(t)] \text{ rotations} \\ &= 360 [R(t) - R_u(t)] \text{ degrees.}\end{aligned}$$

The correction angles for the pass shown in Figure 3 are listed in Table 1. If it is assumed that the magnetic field remained constant in direction during this shadow pass, then the apparent direction would have rotated through the angle $\psi(t)$. It is on this assumption that Figure 2 was plotted. The solid curve shows the azimuthal angle of the interplanetary magnetic field obtained using the pseudo-sun pulse for orientation. The dotted curve shows the relative change $\psi(t)$.

In conclusion, the analysis presented here is an effective method for correcting the azimuthal orientation of the Explorer 35 spacecraft during its passes behind the moon, where it can no longer use the sun as a reference. The accuracy of the method is better than ± 10 degrees, but can only be checked empirically if it is assumed that the magnetic field does not change direction. The largest error would occur during the shadow since, at the beginning and at the end, the orientation has been fitted to the true direction obtained by real-sun sightings.

Table 1
Correction Angles for Pass Shown in Figure 3.

t (sequences)	R(t) (revolutions)	R ₀ (t) (revolutions)	$\psi(t)$ (degrees)
0	0.0	0.0	0
1	35.6697	35.6608	3
2	71.3395	71.3216	6
3	107.0114	106.9823	10
4	142.6856	142.6431	15
5	178.3618	178.2982	23
6	214.0401	213.9532	31
7	249.7203	249.6083	40
8	285.4023	285.2633	50
9	321.0862	320.9184	60
10	356.7717	356.5734	71
11	392.4592	392.2284	83
12	428.1482	427.8835	95
13	463.8386	463.5385	108
14	499.5305	499.1936	121
15	535.2241	534.8486	135
16	570.9187	570.5037	149
17	606.6147	606.1587	164
18	642.3118	641.8138	179
19	678.0103	677.4688	195
20	713.7100	713.1238	211
21	749.4106	748.7789	227
22	785.1123	784.4339	244
23	820.8149	820.0890	216
24	856.5188	855.7440	279
25	892.2234	891.3991	297
26	927.9287	927.0541	315
27	963.6350	962.7092	333
28	999.3425	998.3642	352
29	1035.0503	1034.0192	371
30	1070.7590	1069.6743	391
31	1106.4687	1105.3293	410
32	1142.1787	1140.9844	430
33	1177.8889	1176.7318	417
34	1213.5979	1212.4792	403
35	1249.3059	1248.2266	389

Goddard Space Flight Center
National Aeronautics and Space Administration
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